Structural Macroeconometrics

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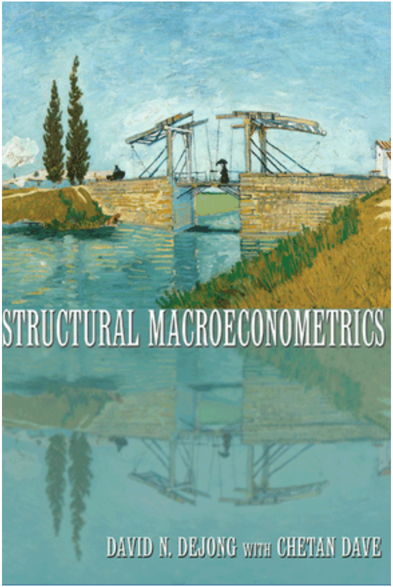

STRUCTURAL MACROECONOMETRICS
DAVID N. DEJONG WITH CHETAN DAVE

Goal of the Course

- Use DSGE models to address empirical issues
- Examples of uses:
  - Model Estimation
  - Model Comparison
  - Forecasting
  - Measurement
  - Shock Identification
  - Policy Analysis

Roots: Systems-of-Equations Models

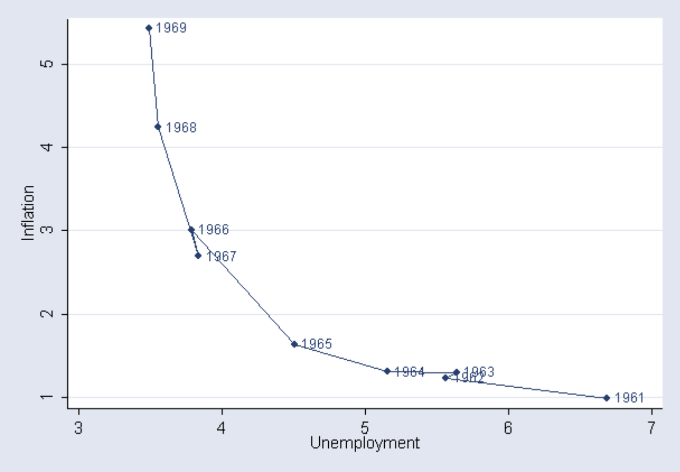
- The representation of theoretical models as complete probability models dates at least to Haavelmo (1944, *Econometrica*)
- Such models consist of:
  - Identities (e.g., Nat'l Income Acc'ting ID)
  - Institutional Rules (e.g., gov't tax policy)
  - Technology Constraints (e.g., production functions)
  - Behavioral Equations (e.g., $C = \overline{C} + (1 - s)YD$)
- Textbook reference: Sargent (1978, Academic Press)

Systems-of-Equations Models, cont.

Classic application: Engineering movements along the (short-run) Phillips Curve.

- Basic Idea: Unemployment is a manifestation of insufficient aggregate demand.
- Remedy: Expansionary fiscal and/or monetary policy.
- Mechanism: Stimulate AD, thus increasing prices, thus decreasing real wages, thus increasing employment.
- Implementation guided by estimated systems-of-equations model.

Case Study: The 1960s Phillips Curve



Seeds of a Revolution

Two developments foreshadowed the demise of the 'systems-of-equations' approach to empirical macro.

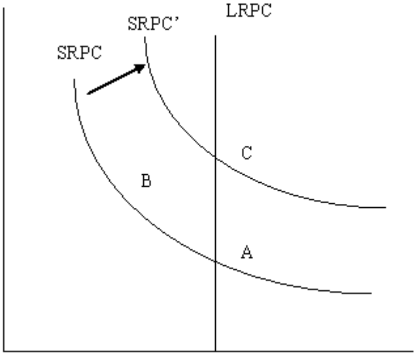
- Theoretical attack on Phillips-curve engineering
- Data

Theory

Phelps (1967, Econometrica), Friedman (1968, AER): Attempts at engineering movements along the SRPC are destabilizing.

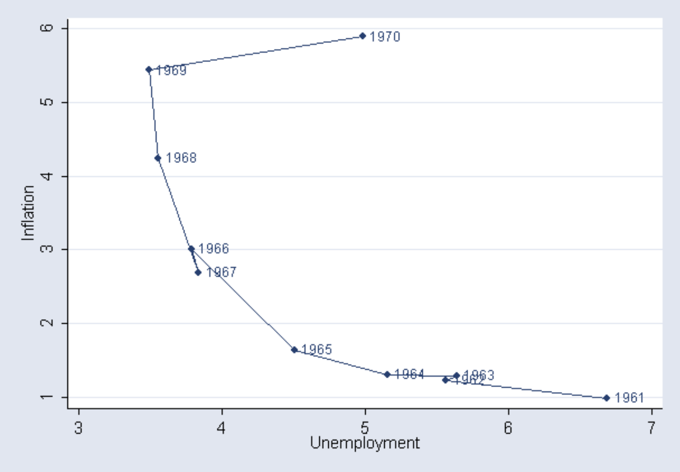
- The creation of AD shocks causes unexpected movements in prices, and thus real wages.
- Once decisionmakers realize that real wages have been shocked, nominal wages will adjust accordingly (adaptive expectations).
- Nominal wage changes (and in general, supply shocks) cause the SRPC to shift.

Theory, cont.



Data

Shift!



Inflation
Unemployment
1970
1969
1968
1966
1967
1965
1964
1963
1961
1
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3
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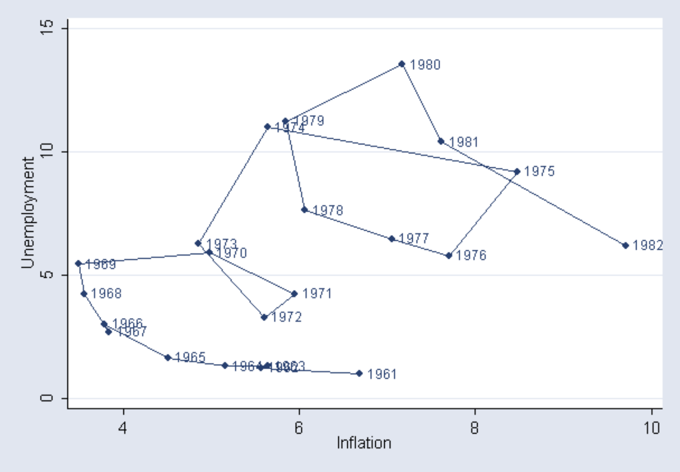
6
7

Data, cont.



Unemployment
Inflation
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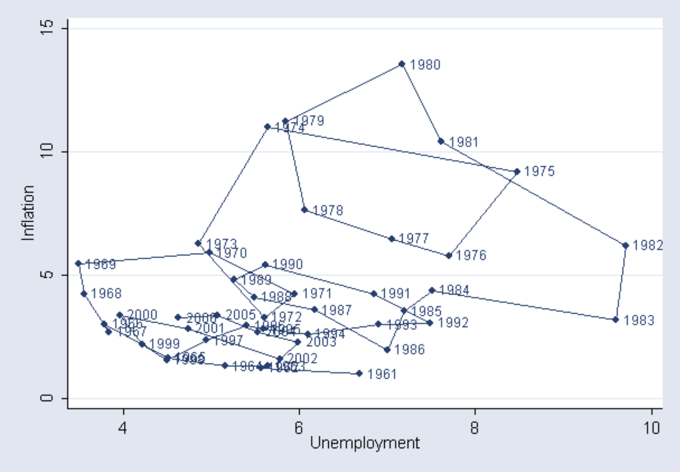
Data, cont.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Bottom Line

Lucas and Sargent (1979, FRB Minneapolis QR):

"In the present decade, the U.S. economy has undergone its first major depression since the 1930's, to the accompaniment of inflation rates in excess of 10 percent per annum.... These events ... were accompanied by massive government budget deficits and high rates of monetary expansion, policies which, although bearing an admitted risk of inflation, promised according to modern Keynesian doctrine rapid real growth and low rates of unemployment. That these predictions were wildly incorrect and that the doctrine on which they were based is fundamentally flawed are now simple matters of fact, involving no novelties in economic theory." [p. 1]

The Revolution: Lucas' Critique

The Problem

Lucas (1976 Carnegie-Rochester Series on Public Policy):
"... given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models." [p. 41]

Lucas' Critique, cont.

Implication for 'systems-of-equations models'

Lucas (1976):
"... simulations using these models can, in principle, provide no useful information as to the actual consequences of alternative economic policies." [p. 20]

Lucas and Sargent (1979):
"... the difficulties [with these models] are *fatal*: that modern macroeconomic models are of *no* value in guiding policy and that this condition will not be remedied by modifications along any line which is currently being pursued." [p. 2]

Implementing Structural Models Empirically

- Indirect Implementation
  - Cross-Equation Restrictions
  - Identified VARs
  - Inducing Priors Using DSGEs
- Direct Implementation
  - Calibration
  - Matching Moments
  - Likelihood representations

Indirect Implementation 1: Cross-Equation Restrictions

Sims (1972 AER), Hansen and Sargent (1980 JEDC)

By embedding a VAR into a structural model, under the assumption that decision makers form expectations through the use of VARs, one can derive restrictions across the equations of the VAR, thus imposing theoretical restrictions on these otherwise flexible reduced-form models.

Cross-Equation Restrictions, cont.

- Drawback: Cross-equations restrictions tests routinely yield rejections
- Examples:
  - Term structure of interest rates (Hansen and Sargent, 1981 *FRB Minneapolis Staff Report*)
  - Consumption-based asset-pricing models (Hansen and Singleton, 1982 *Econometrica*; 1983 *JPE*)
  - Inventory adjustment (Eichenbaum, 1983 *JME*)
- Lucas (1980, *JMCB*): "Any model that is well enough articulated to give clear answers to the questions we put to it will necessarily be artificial, abstract, patently 'unreal'." [p. 696]

Indirect Implementation 2: Identified VARs.

- Goal: Assign structural interpretation to reduced-form innovations
- Sims (1980 *Econometrica*): Decompositions of innovation VCV matrix
- Blanchard and Quah (1989 *AER*), King, Plosser, Stock and Watson (1991 *AER*): long-run restrictions on impulse response functions
- Fernandez-Villaverde, Rubio-Ramirez and Sargent (2006 *Econometrica*): Impose identifying restrictions using DSGE models

Indirect Implementation 3: Using DSGE Models to Induce Priors Over VARs

- Goal: Obtain 'weak' restrictions over VARs
- Approach: exploiting mapping from structural parameters μ to reduced-form parameters θ, use $\pi(\mu)$ to construct $\pi(\theta)$.
- DeJong, Ingram and Whiteman (1993 *AEA Proceedings*); Ingram and Whiteman (1994 *JME*); Sims and Zha (1998 *IER*); Del Negro and Schorfheide (2004, *IER*).

Direct Implementation, 1: Calibration

Kydland and Prescott (1982 Econometrica) [Historical roots date to Frisch, 1933 Econometrica]

K&P called for an abandonment of the probabilistic approach to econometrics, which they equated to hypothesis testing.

Prescott (1986 *FRB Minneapolis QR*):
"The models constructed within this theoretical framework are necessarily highly abstract. Consequently, they are necessarily false, and statistical hypothesis testing will reject them. This does not imply, however, that nothing can be learned from such quantitative theoretical exercises." [p. 10]

Calibration, cont.

Implementation (Kydland and Prescott, 1996 JEP; Prescott, 2006 JPE)

1. Pose a question (two general types: measured impacts of policy changes; fit)
2. Use "well-tested theory" to address the question
3. Construct a model economy
4. Calibrate: "Generally, some economic questions have known answers, and the model should give an approximately correct answer to them if we are to have any confidence in the answer given to the question with unknown answer. Thus, data are used to calibrate the model economy so that it mimics the world as closely as possible along a limited, but clearly specified, number of dimensions." [p. 74]
5. Run the experiment

Calibration, cont.

Drawback:

The lack of statistical formality associated with calibration exercises imposes distinct limitations upon what can be learned and communicated via their use.

Calibration, cont.

Haavelmo (1944 Econometrica):

"So far, the common procedure has been, first to construct an economic theory involving *exact* functional relationships, then to compare this theory with some actual measurements, and, finally, "to judge" whether the correspondence is "good" or "bad." Tools of statistical inference have been introduced, in some degree, to support such judgements, e.g., the calculation of a few standard errors and multiple-correlation coefficients....

Calibration, cont.

Continuing with Haavelmo (1944 Econometrica):

"... The application of such simple "statistics" has been considered legitimate, while, at the same time, the adoption of definite probability models has been deemed a crime in economic research, a violation of the very nature of economic data. That is to say, it has been considered legitimate to use some of the *tools* developed in statistical theory *without* accepting the very *foundation* upon which statistical theory is built. For *no tool developed in the theory of statistics has any meaning* - except, perhaps, for descriptive purposes - *without being referred to some stochastic scheme*." [p. iii]

Direct Implementation 2: Matching Moments

- Idea: estimate model parameters as those that provide the best fit to a prespecified set of moments.
- Aids communication by re-introducing statistical formality.
- Caveat: inferences can be sensitive to choice of moments.
- GMM (Hansen, 1982 *Econometrica*)
- SMM (McFadden, 1986 *Econometrica*; Pakes and Pollard, 1989 *Econometrica*; Lee and Ingram, 1991 *JoE*; Duffie and Singleton, 1993 *Econometrica*.)
- Indirect Inference (Gourieroux, Monfort and Renault, 1993 *JAE*; Smith, 1993 *JAE*.)

Direct Implementation 3: Likelihood Representations of Linear Approximations

Sargent (1989 JPE):
Established the mapping of linearized DSGE models into state-space representations. Under the assumption of normality for stochastic innovations and measurement errors, associated likelihood functions can be evaluated via the Kalman filter.

DeJong, Ingram and Whiteman (2000 JoE; 2000 JAE):
Developed methods for mapping priors over structural parameters into priors over corresponding state-space parameters, enabling Bayesian inference.

Direct Implementation, 4: Likelihood Representations of Non-Linear Approximations

Fernandez-Villaverde and Rubio-Ramirez (2005 JAE; 2007 REStud):

- Problem: Second-order model approximation errors accumulate on a period-by-period basis into first-order errors associated with corresponding likelihood approximations.
- Remedy: Use (relatively accurate) non-linear model approximations, achieve likelihood evaluation via use of the Particle Filter.

Non-Linear Approximations, cont.

DeJong, Dharmarajan, Liesenfeld, Richard (2007 WP):

- Problem: The Particle Filter is numerically inefficient.
- Remedy: EIS Filter

Course Outline

- Model Solution
  - Converting environments into non-linear expectational difference equations (Dynamic Programming)
  - Linear approximation
  - Non-Linear approximation
- Likelihood Evaluation
  - State-Space Representations
  - The Kalman Filter
  - Importance Sampling
  - The Particle Filter
  - Adaptation
  - The EIS Filter

Notation

- The steady state value of y_t is denoted as $\overline{y}$
- Logged deviations of variables from steady state values are denoted using tildes; e.g., $\widetilde{y}_t = \log\left(\frac{y_t}{\overline{y}}\right)$
- The vector x_t denotes the collection of model variables; e.g., $x_t = [\widetilde{y}_t \quad \widetilde{c}_t \quad \widetilde{n}_t]'$
- The vector υ_t denotes the collection of structural shocks incorporated in the model.
- The vector η_t denotes the collection of expectational errors associated with intertemporal optimality conditions.

Notation, cont.

- Log-linear approximations of structural models are represented as

$$Ax_{t+1} = Bx_t + Cv_{t+1} + D\eta_{t+1}, \quad (1)$$

where the elements of the matrices A, B, C and D are functions of the structural parameters μ.

- Solutions of (1) are expressed as

$$x_{t+1} = F(\mu)x_t + G(\mu)v_{t+1}. \quad (2)$$

- Observable variables are denoted by X_t, where

$$X_t = H(\mu)'x_t + u_t, \quad (3)$$

where $E(u_t u_t') = \Sigma_u$. The presence of u_t in (3) reflects the possibility that the observations of X_t are associated with measurement error.

- Defining $e_{t+1} = G(\mu)v_{t+1}$, the covariance matrix of e_{t+1} is given by

$$Q(\mu) = E(e \cdot e') \quad (4)$$

Notation, cont.

- Non-linear approximations of structural models are represented using two equations.
- The first characterizes the evolution of the state variables s_t (a subset of the full collection of variables contained in x_t):

$$s_t = f(s_{t-1}, \upsilon_t), \tag{5}$$

  where once again υ_t denotes the collection of structural shocks incorporated in the model.
- The second maps the state variables into the observables:

$$X_t = g(s_t, u_t), \tag{6}$$

  where once again u_t denotes measurement error.
- Associated likelihood function:

$$L(X|\mu).$$